Proving Einsteins Theory of Relativity with Optics

Author: David Eduardo Toro

Affiliations:[1] Florida Institute of Technology, Melbourne, Florida (USA)

Abstract: We prove that Einstein's Theory of Relativity is the reason that light gets refracted in differing mediums (as proven by Snells Law) in optical systems. Theoretical and experimental results are in agreement.

$$n_{air} = 1.00029$$
$$n_{diamond} = 2.42$$
$$c = 299{,}792{,}458 m/s$$
$$v_{diamond} = 123{,}881{,}181 m/s$$
$$v_{air} = 299{,}705{,}543.4 m/s$$

If we choose an arbitrary frequency, such as the frequency of the blue light, the wavelength in the air, is as follows:

$$f_{blue} = 6.5 \cdot 10^{14} Hz$$
$$\lambda_{air} = \left(\frac{299{,}705{,}543.4 m/s}{6.5 \cdot 10^{14} Hz} \right) = 461.085 nm$$

We can calculate the frequency of light inside of the diamond by using Einstein's Mass Dilation equation. Since the velocity changes inside of the diamond, and this causes relativistic effects, then we can calculate the new mass.

The following is the rest mass equation as developed by Albert Einstein [1].

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_o c}{\sqrt{c^2 - v^2}}$$

However, since the velocity of light was not initially at rest in the air we need to add the following term.

$$m = \frac{m_o c}{\sqrt{c^2 - (v_{diamond} - v_{air})^2}}$$

To plug in all the numbers, first, we need to calculate the rest mass of the light in the air.

$$h = 6.62607004 \cdot 10^{-34} m^2 kg / s$$

$$m_0 v_{air}^2 = hf$$

$$m_0 = \frac{hf}{v_{air}^2}$$

$$m_o = \frac{hf_{blue}}{v_{air}{}^2} = \frac{\left(6.62607004 \cdot 10^{-34} m^2 kg / s\right)\left(6.5 \cdot 10^{14} Hz\right)}{\left(299,705,543.4 m / s\right)^2} = 4.794903015 \cdot 10^{-36} kg$$

Now we can calculate the mass of the light in the Diamond.

$$m_o c = (4.794903015 \cdot 10^{-36} kg)\left(299792458 m / s\right) = 1.437475761 \cdot 10^{-27} kg \cdot m / s$$

$$\sqrt{c^2 - (v_{diamond} - v_{air})^2} = \sqrt{(299792458)^2 - (123,881,181 - 299,705,543.4)^2} = 242819503.9 m / s$$

$$m = \frac{m_o c}{\sqrt{c^2 - (v_{diamond} - v_{air})^2}} = 5.919935335 \cdot 10^{-36} kg$$

Now to get the frequency of light inside of the diamond, we use the mass in the diamond and the velocity of light in the diamond.

$$f_{diamond} = \frac{m v_{diamond}^2}{h} = \frac{(5.919935335 \cdot 10^{-36} kg)(123,881,181 m / s)^2}{(6.62607004 \cdot 10^{-34} m^2 kg / s)} 1.371107841 \cdot 10^{14} Hz$$

The wavelength of the light in the Diamond is:

$$\lambda = \frac{v_{diamond}}{f_{diamond}} = \frac{123,881,181 m / s}{1.371107841 \cdot 10^{14} Hz} = 90.4 \mu m$$

In this section, we can calculate the time it takes to slow down the light and the distance the light travels so it could begin moving at a constant velocity.

$$f_{blue} = 6.5 \cdot 10^{14}\,Hz$$

The time it takes for light to travel one cycle before the time gets dilated.

$$t_{stationary} = \frac{1}{f_{blue}} = \frac{1}{6.5 \cdot 10^{14}\,Hz} = 1.538461538 \cdot 10^{-15} s$$

$$c = 299{,}792{,}458 m/s$$

$$v_{air} = v_{stationary} == 299{,}705{,}543.4 m/s$$

$$v_{diamond} = v_{moving} = 123{,}881{,}181 m/s$$

Time dilation is responsible for the deceleration of lights velocity when it enters into the diamond as follows:

$$a = \frac{v_{moving} - v_{stationary}}{t} = \frac{\left(v_{moving} - v_{stationary}\right)c}{t_{stationary}\left(\sqrt{c^2 - \left(v_{moving} \pm v_{stationary}\right)^2}\right)}$$

To solve the following we can plug in all the numbers

$$t_{moving} = \frac{t_{stationary}}{c}\left(\sqrt{c^2 - \left(v_{moving} \pm v_{stationary}\right)^2}\right)$$

The time is dilated and this corresponds to the time it takes to slow the light. Because once the light moving at the velocity of air reaches the speed in the diamond the time dilation stops.

$$v_{moving} - v_{stationary} = 123{,}881{,}181 m/s - 299{,}705{,}543.4 m/s = -175{,}824{,}362.4 m/s$$

$$t_{moving} = \frac{\left(1.538461538 \cdot 10^{-15} s\right)\sqrt{(299{,}792{,}458 m/s)^2 - (123{,}881{,}181 m/s - 299{,}705{,}543.4 m/s)^2}}{299{,}792{,}458 m/s}$$

$$t_{moving} = 1.246090278 \cdot 10^{-15} s$$

Plugging in the numbers we get the following deceleration of the light:

$$a = \frac{v_{moving} - v_{stationary}}{t} = \frac{\left(v_{moving} - v_{stationary}\right)c}{t_{stationary}\left(\sqrt{c^2 - \left(v_{moving} - v_{stationary}\right)^2}\right)}$$

$$a = -1.411008219 \cdot 10^{23} m/s^2$$

Thus, if we get the average velocity of the light inside of the diamond.

$$v_{avg} = \left(\frac{v_{moving} + v_{stationary}}{2}\right) = \left(\frac{123{,}881{,}181 m/s + 299{,}705{,}543.4 m/s}{2}\right) = 211{,}793{,}362.2 m/s$$

Thus the distance it takes for light to slow down inside of the diamond is:

$$x = v_{avg} t_{moving} - \frac{1}{2} a t_{moving}^2$$

$$= \left(211{,}793{,}362.2 m/s\right)\left(1.246090278 \cdot 10^{-15} s\right) - 0.5\left(1.411008219 \cdot 10^{23} m/s^2\right)\left(1.246090278 \cdot 10^{-15} s\right)^2$$

$$= 154.367 nm$$

Now we can derive the frequency of the light as it exits the diamond. With this, we can prove that the incident light velocity is the same as the velocity when the light exits the diamond as predicted by Snells Law.

First, we need to calculate the rest mass of light inside of the diamond.

$$m_o = \frac{hf_{diamond}}{v_{diamond}^2} = \frac{\left(6.62607004 \cdot 10^{-34} m^2 kg/s\right)\left(1.371107841 \cdot 10^{14} Hz\right)}{\left(123{,}881{,}181 m/s\right)^2} = 5.919935334 \cdot 10^{-36} kg$$

$$v_{diamond} = 123{,}881{,}181 m/s$$

$$v_{air} = 299{,}705{,}543.4 m/s$$

Again, the mass gets dilated when it exits the diamond due to the change in velocity of the light from the denser diamond medium to the air medium.

$$m = \frac{m_o c}{\sqrt{c^2 - (v_{air} - v_{diamond})^2}}$$

$$m_o c = \left(5.919935334 \cdot 10^{-36} kg\right)\left(299{,}792{,}458 m/s\right) = 1.774751965 \cdot 10^{-27} kg \cdot m/s$$

$$\sqrt{c^2 - (v_{air} - v_{diamond})^2} = \sqrt{(299{,}792{,}458)^2 - (299{,}705{,}543.4 - 123{,}881{,}181 m/s)^2} = 242{,}819{,}503.9 m/s$$

The mass of the light in the air is:
$m = 7.308934977 \cdot 10^{-36}\, kg$

The frequency of light outside of the Diamond is:

$$f_{air-outside-of-diamond} = \frac{m \cdot v_{air}^2}{h} = \frac{\left(7.308934977 \cdot 10^{-36}\, kg\right)\left(299{,}705{,}543.4 m/s\right)^2}{\left(6.62607004 \cdot 10^{-34}\, m^2 kg/s\right)} = 9.908037181 \cdot 10^{14}\, Hz$$

Now we can determine the lamda of the light emitted from the diamond. And it's a well-known fact that blue light makes diamonds look darker because this frequency of incident light produces infrared light as it exits the diamond.

$$\lambda = \frac{v_{air}}{f_{air-outside-of-diamond}} = \frac{\left(299{,}705{,}543.4 m/s\right)}{\left(9.908037181 \cdot 10^{14}\, Hz\right)} = 30.2 \mu m$$

We can calculate the acceleration of the light to prove that the light gets re-accelerated as predicted in optics.

The time to complete a cycle for the light inside the diamond is:

$$t_{diamond} = \frac{1}{f_{diamond}} = \frac{1}{(1.371107841 \cdot 10^{14}\, Hz)} = 7.293372338 \cdot 10^{-15} s$$

Using the rest mass time we can compute the time dilated time which will create the positive acceleration as the light exits the diamond.

$$t_{outside} = \frac{t_{diamond}\sqrt{c^2 - (v_{air} - v_{diamond})^2}}{c}$$

$$= \frac{\left(7.293372338 \cdot 10^{-15} s\right)\sqrt{(299{,}792{,}458)^2 - (299{,}705{,}543.4 - 123{,}881{,}181 m/s)^2}}{(299{,}792{,}458)}$$

$$= 5.907330239 \cdot 10^{-15} s$$

$$a = \frac{\left(v_{air} - v_{diamond}\right)}{t_{outside}} = \frac{\left(299{,}705{,}543.4 m/s - 123{,}881{,}181 m/s\right)}{\left(5.907330239 \cdot 10^{-15} s\right)}$$

$$a = 2.976376049 \cdot 10^{22}\, m/s^2$$

$$v_{diamond} = 123{,}881{,}181 m/s$$

Thus the velocity as the light exits the diamond is as expected: the speed of light in air!

$$v_{outside} = v_{diamond} + a \cdot t_{outside}$$

$$v_{outside} = 123,881,181 m/s + \left(2.976376049 \cdot 10^{22} m/s^2\right)\left(5.907330239 \cdot 10^{-15} s\right)$$

$$= 299,705,543.4 m/s$$

Thus the distance to accelerate the light from the speed of light in diamond to that of air is:

$$x = v_{diamond} t_{outside} + \frac{1}{2} a t_{outside}^2$$

$$x = \left(123,881,181 m/s\right) \cdot \left(5.907330239 \cdot 10^{-15} s\right) + \frac{1}{2}\left(2.976376049 \cdot 10^{22} m/s^2\right) \cdot (5.907330239 \cdot 10^{-15} s)^2$$

=1251nm

Here is a recap of our results:

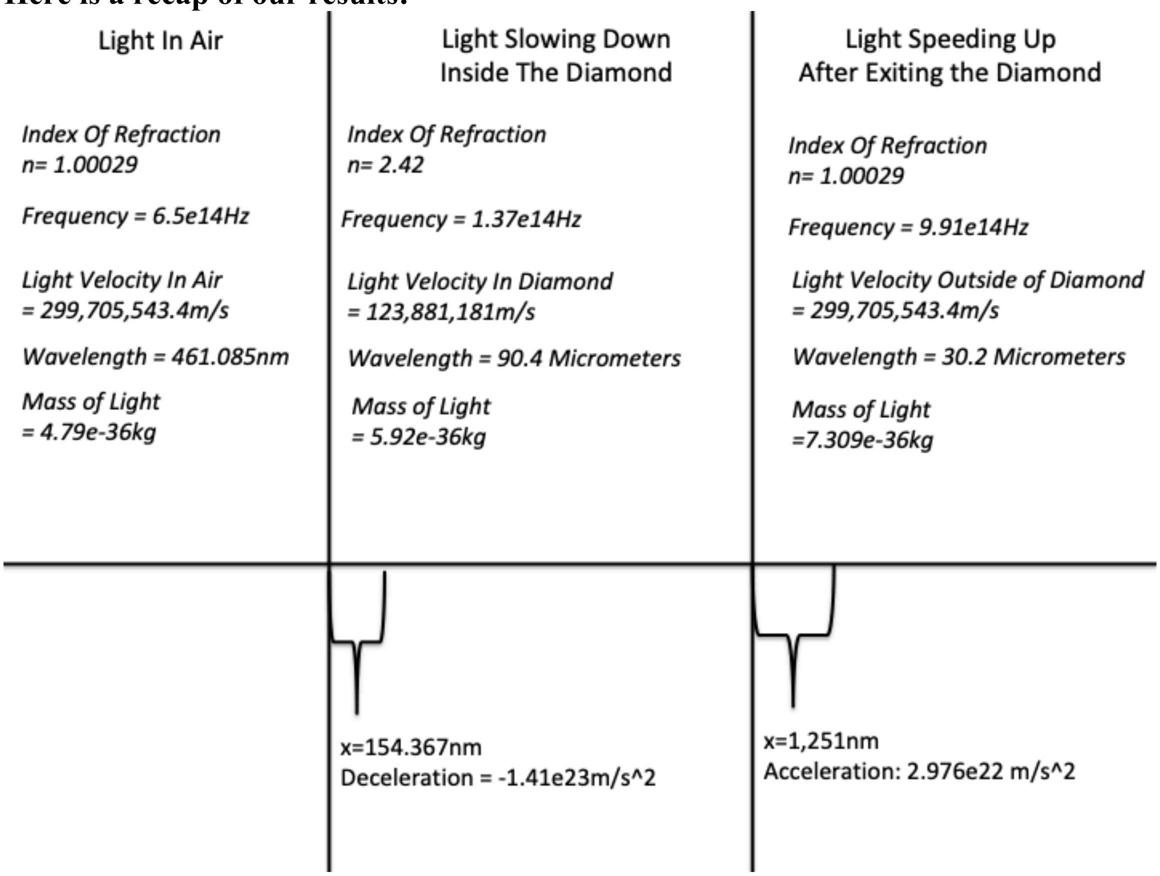

Fig. 1: Recap of the Data Analysis of the Incident Light Ray, The Light in Diamond, and the Light exiting the diamond.

References:

1. Albert Einstein, 1905, "On the Electrodynamics of Moving Bodies".